Note

Flow in Elliptical Channels

THE ANALYSIS OF SHEAR-VISCOUS FLOW IN "TYPICAL" ELLIPTICAL DIES

The general treatment of rectangular dies (loc. cit.) has yielded the following equation for parallel-sided rectangular dies:

$$\Delta P = 2\eta L(\xi + 1)\, \xi^{-(n+1)} \left(Q \frac{4n + 2}{n}\right)^n H^{-(3n+1)} \tag{1}$$

where ξ represents the ratio of W/H of the cross section. We now consider a "typical" elliptical die of equal cross-sectional area in which the ratio of the major and minor axes A/B is likewise equal to ξ. A "typical" ellipse is defined as an ellipse which is sufficiently eccentric for the isovels in laminar flow *not* to be approximated by those of a circle, but not squashed to such a degree that the isovels may be taken to exhibit a pattern which closely resembles that of flow in a wide-slit die. Taking ξ as a parameter of eccentricity, we define a "typical" ellipse arbitrarily as one in which $10 \geqslant \xi \geqslant 1.25$.

The area of an ellipse is given by

$$\pi AB = \pi \xi B^2$$

and its circumference by

$$\pi(A + B) = \pi B(\xi + 1)$$

where A and B represent the lengths of the half-axes. From the balance of forces during steady-state pressure flow, the shear stress τ is therefore found to be

$$\tau = \frac{\Delta P \xi B}{L(\xi + 1)} \tag{2}$$

We now make the reasonable assumption that the pattern of isovels in flow through a typical elliptical channel as defined above is practically identical with that which exists in a rectangular channel of equal cross-sectional area and in which $\xi = W/H = A/B$, so that we may write

$$\dot{\gamma} \simeq \frac{Q}{\xi H^3} \cdot \frac{4n + 2}{n} \tag{3}$$

which embodies the Rabinowitsch correction for rectangular flow geometries. Equating the area of the rectangle

$$WH = \xi H^2$$

with that of the ellipse, it is seen that $H = B\sqrt{\pi}$.

Substituting in eq. (2),

$$\dot{\gamma} = (B\sqrt{\pi})^{-3}\, \xi^{-1} \left(Q \cdot \frac{4n + 2}{n}\right)$$

and using the Power law,

$$\eta = \frac{\tau}{\dot{\gamma}^n} = \frac{\Delta P}{L(\xi + 1)} \xi^{n+1}\, \pi^{3n/2} \left(Q \cdot \frac{4n + 2}{n}\right)^{-n} B^{3n+1}$$

Making ΔP the subject of the equation, we obtain

$$\Delta P = \eta L(\xi + 1)\xi^{-(n+1)} \left(\frac{Q}{\pi^{3/2}} \cdot \frac{4n + 2}{n}\right)^n B^{-(3n+1)} \tag{4}$$

In the Newtonian case ($n = 1$),

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$$\Delta P = \eta L(\xi + 1)\xi^{-2}\frac{6Q}{\pi^{3/2}}B^{-4}$$

and for $n = 1/3$ (a common value for many polymers),

$$\Delta P = \eta L(\xi + 1)\xi^{-4/3}\frac{(10Q)^{1/3}}{\pi^{1/2}}B^{-2}$$

We now consider linearly converging elliptical channels of taper angle θ and a constant shape defined by $A/B = a/b = \xi$. Converting equation (4) to the incremental pressure drop dP

$$dP = \eta\, dl(\xi + 1)\xi^{-(n+1)}\left(\frac{Q}{\pi^{3/2}}\cdot\frac{4n+2}{n}\right)^n b^{-(3n+1)}$$

and substituting for dl (from trigonometry),

$$dl = -\frac{db}{2}\cot\theta$$

we obtain

$$dP = -\eta\cot\theta(\xi + 1)\xi^{-(n+1)}\left(\frac{Q}{\pi^{3/2}}\cdot\frac{4n+2}{n}\right)^n b^{-(3n+1)}$$

Integration between the limits of B_1 and B_2, the minor half-axis at the die entrance and the die exit respectively, then gives

$$\Delta P = \frac{\eta\cot\theta}{3n}(\xi + 1)\xi^{-(n+1)}\left(\frac{Q}{\pi^{3/2}}\cdot\frac{4n+2}{n}\right)^n (B_2^{-3n} - B_1^{-3n}) \quad (5)$$

For a Newtonian, this reduces to

$$\Delta P = \frac{4}{\pi^{3/2}}Q\eta\cot\theta$$

And for $n = 1/3$ (a common value for many polymers), we obtain

$$\Delta P = \eta\cot\theta(\xi + 1)\xi^{-4/3}\frac{(10Q)^{1/3}}{\pi^{1/2}}\left(\frac{1}{B_2} - \frac{1}{B_1}\right)$$

DISCUSSION

It should be pointed out, again, that the approximation used is only applicable to "typical" ellipses as defined above. If $A = B = R$ (circle, $\xi = 1$) then eq. (4), when compared with the appropriate equation for the pressure drop in parallel-sided circular dies, shows an error of about 15% when $n = 1$ and an error of about 6% when $n = 1/3$. On the other hand, a comparison of eq. (4) and that which applies to wide-slit dies (ξ very large) establishes an exact identity. It all depends, however, on what is understood to be "very large," so that $\xi + 1$ may be taken to be equal to ξ. Clearly, a value of 10, arbitrarily chosen, will not be large enough to rule out an error of a few percent, but it is argued that this is acceptable for all practical purposes.

SUMMARY

The article presents an analysis for the flow of power-law fluids through linearly tapering and parallel-sided elliptical dies based upon the approximation that the flow pattern in such dies closely resembles that which characterizes laminar flow in rectangular channels of equal cross-sectional area and equal shape factor ξ, where $\xi = W/H$ (rectangle) $= A/B$ (ellipse). This analysis is of particular value for "typical" ellipses, i.e., for ellipses that are sufficiently eccentric to render any circular approximation inaccurate and that are not squashed to such an extent as to make the wide-slit approximation the obvious choice of the assumed flow geometry.

REFERENCES

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